Quiz 4 Solution

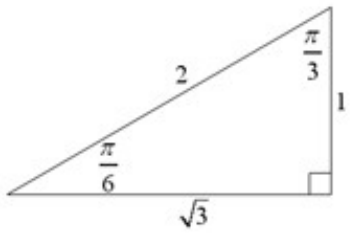
September 11, 2017

1. (2 points) Find the derivative of $y = (3\cos x + 2\tan x)\sin x$ at $x = \frac{\pi}{6}$. Leave your answer **exact**.

Solution:
First, we need to find y'. We use Product Rule:

$$\begin{aligned} y' &= \frac{d}{dx}[(3\cos x + 2\tan x)]\sin x + (3\cos x + 2\tan x)\frac{d}{dx}[\sin x] \text{ by Product Rule} \\ &= (-3\sin x + 2\sec^2 x)\sin x + (3\cos x + 2\tan x)\cos x \text{ by taking derivatives} \end{aligned}$$

Since we're evaluating at a point, don't worry about simplifying. Instead, plug in $\frac{\pi}{6}$ for x. We can find the trig functions at $\frac{\pi}{6}$ from the triangle



$$\begin{aligned} y'(\frac{\pi}{6}) &= (-3\sin\frac{\pi}{6} + 2\sec^2\frac{\pi}{6})\sin\frac{\pi}{6} + (3\cos\frac{\pi}{6} + 2\tan\frac{\pi}{6})\cos\frac{\pi}{6} \\ &= \left(-3\left(\frac{1}{2}\right) + 2\left(\frac{2}{\sqrt{3}}\right)^2\right)\left(\frac{1}{2}\right) + \left(3\left(\frac{\sqrt{3}}{2}\right) + 2\left(\frac{1}{\sqrt{3}}\right)\right)\left(\frac{\sqrt{3}}{2}\right) \\ &= \left(\frac{-3}{2} + \frac{8}{3}\right)\left(\frac{1}{2}\right) + \left(\frac{3\sqrt{3}}{2} + \frac{2}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{2}\right) \\ &= \left(\frac{-3}{4} + \frac{8}{6}\right) + \left(\frac{9}{4} + 1\right) \text{ by distributing} \\ &= \frac{-9}{12} + \frac{16}{12} + \frac{27}{12} + \frac{12}{12} \text{ by finding a common denominator} \\ &= \frac{54}{12} = \frac{23}{6} \end{aligned}$$

Answer: 23/6

2. (2 points) Given $f(x) = \dfrac{9\cot x}{6 + 4\sec x}$, find $f'(x)$. Do **not** simplify.

Solution: We use Quotient Rule.

$$\begin{aligned} f'(x) &= \frac{(6 + 4\sec x)\frac{d}{dx}[9\cot x] - 9\cot x\frac{d}{dx}[6 + 4\sec x]}{(6 + 4\sec x)^2} \\ &= \frac{(6 + 4\sec x)(-9\csc^2 x) - 9\cot x(4\sec x\tan x)}{(6 + 4\sec x)^2} \end{aligned}$$

Don't simplify any further than this.

Answer:

$$\frac{(6 + 4\sec x)(-9\csc^2 x) - 9\cot x(4\sec x\tan x)}{(6 + 4\sec x)^2}$$

3. (1 point) What do you want to review on Friday?

Answer: Answers will vary.